Propagating Uncertainty Using GUM Supplement 1

Presented at the 2006 TLD and Records Symposium

References

- **BIPM/IEC/IFCC/ISO/IUPAC/IUPAP/OIML:1993**, Guide to the Expression of Uncertainty in Measurement (GUM)
- **ISO/PRF Guide 99998**; Guide to the expression of uncertainty in measurement (GUM) -- Supplement 1: Numerical methods for the propagation of distributions, 2004
- "**Evaluating the Uncertainty in Measurement of Occupational Exposure with Personal Dosemeters**"; J.W.E. van Dijk, NRG Radiation & Environment, presented at IM2005, Vienna Austria.
- **ISO/IEC 17025:1999**, General requirements for the competence of testing and calibration laboratories
- **NIST Technical Note 1297**; Guidelines for Evaluating and Expressing the Uncertainty of NIST Measurement Results, 1994.
- **NCRP Report No. 58**, A Handbook of Radioactivity Measurements Procedures, 1985.
- Hirning, C. R. and Yuen P. S., "**Accuracy in External Dosimetry of Ionizing Radiation**," Health Physics 75(2) pp 136-146, 1998.

Overview

- Goal
  - Estimate total uncertainty in calculated dose results based on assumed uncertainties in element response
  - *i.e.- propagate uncertainties through algorithm for a particular dose result*
- Method
  - Estimate element uncertainties
  - Generate sample of "random" responses
  - Process sample through PC algorithm
  - Calculate statistics on resulting distribution of calculated doses

Uncertainty in TL Dosimetry

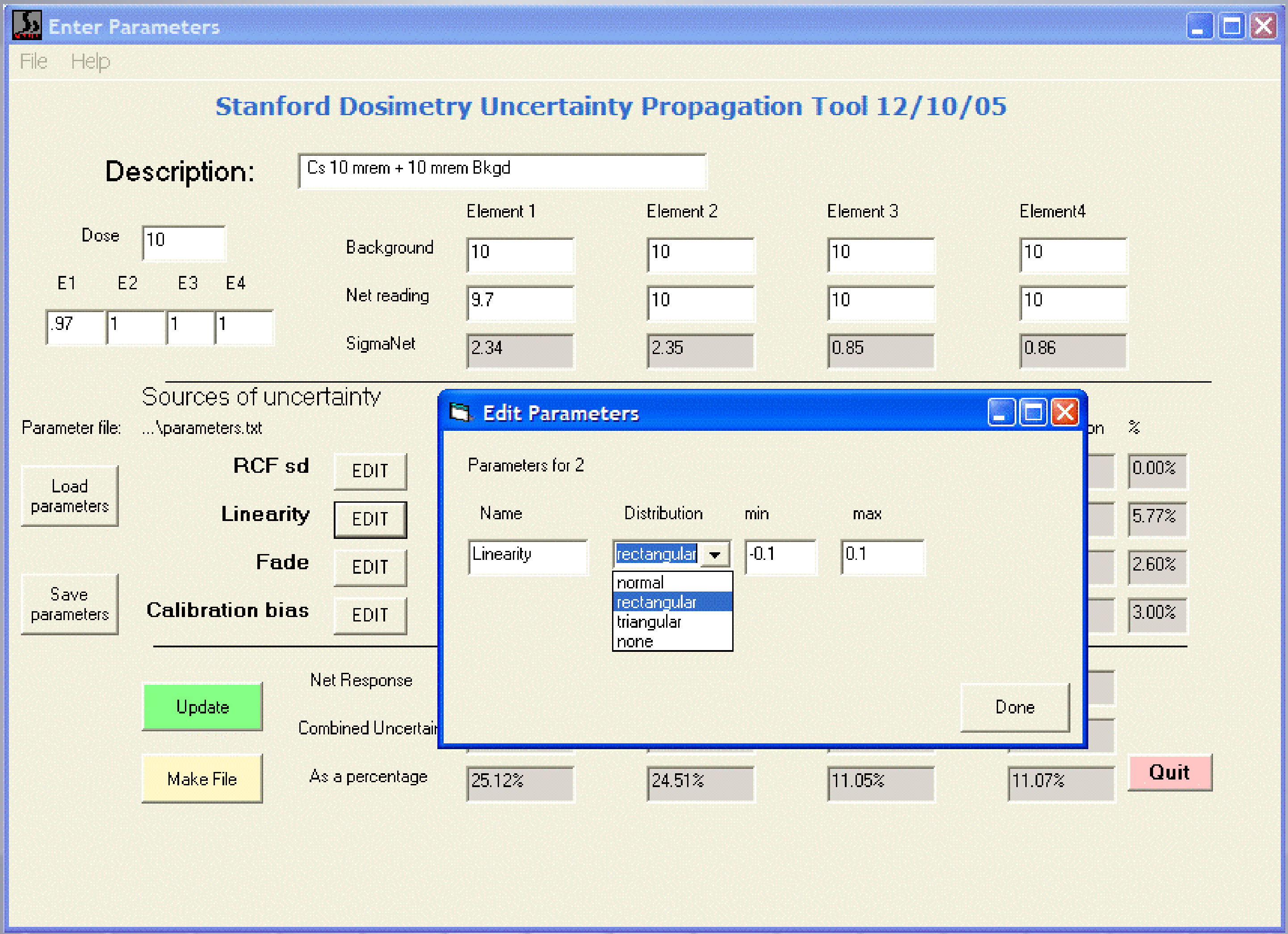
- Identify sources
- Measure or estimate individual uncertainties
- Calculate combined uncertainty in element responses
- Calculate total propagated uncertainty in final dose result

Sources of Uncertainty
- Example – Panasonic UD-802

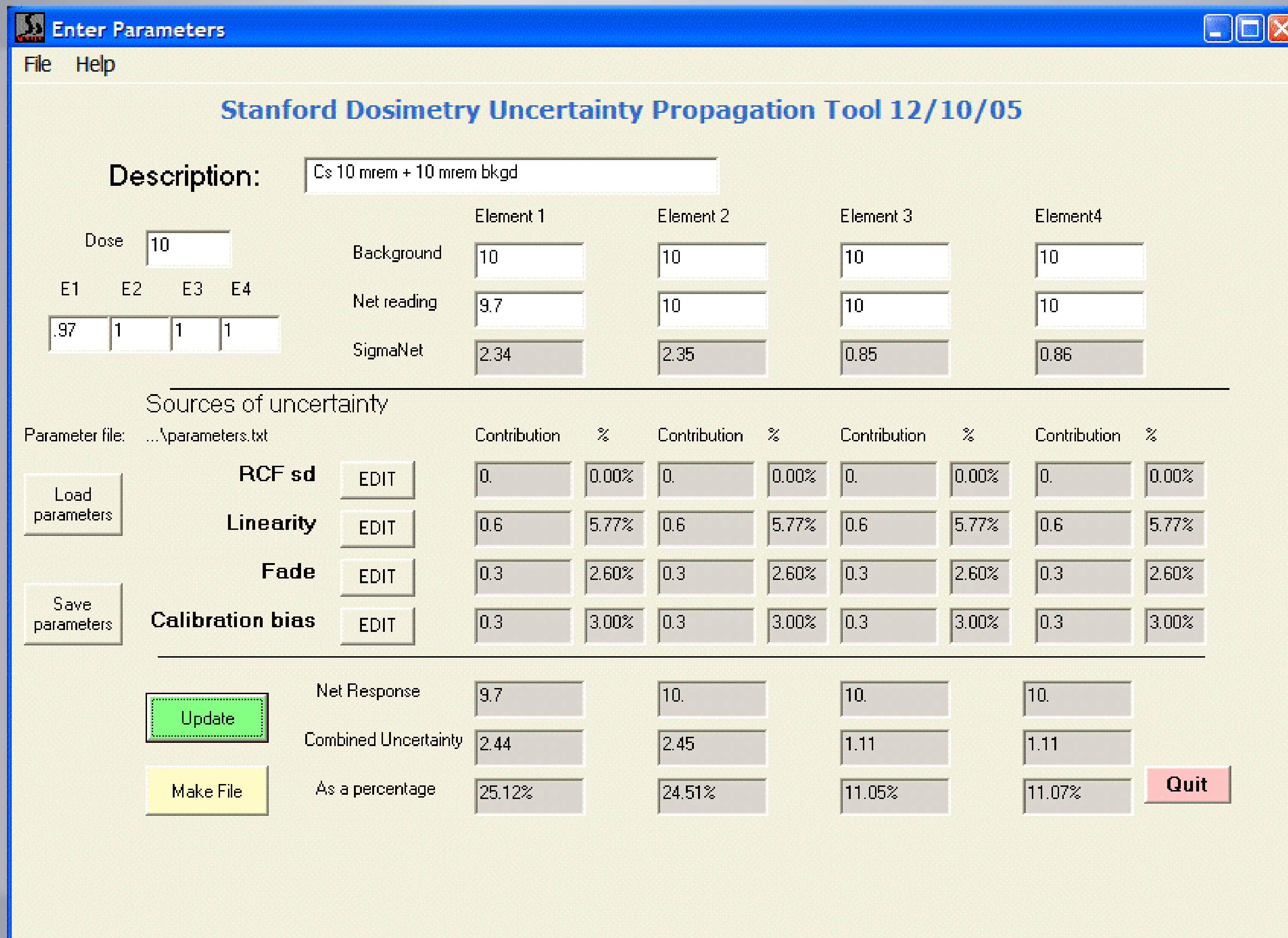
Source	A/B	Source	Distribution	Method of est.	Range	Value
Random uncertainty in TLD reading	A	data	Normal	Model u vs. reading	N/A	
Reader calibration	B	procedure	Normal	3% sd	± 9%	reading*.03
Reader linearity	B	Panasonic	Rectangular	[Max. range/2]/√3	± 10% of reading	reading*.058
TL Fade ***accuracy of correction***	B	test	Rectangular	[Max. range/2]/√3	-2% to +7% of reading	reading*.026

Distribution type and estimate of standard uncertainty based on guidance in GUM (or NIST 1297)

Calculate Combined Uncertainty



Calculate Combined Uncertainty



Total Propagated Uncertainty

What about the algorithm?

How do we determine the impact of element uncertainties on the final dose?

Option 1 - Law of propagation of uncertainty (GUM and NIST TN1287)

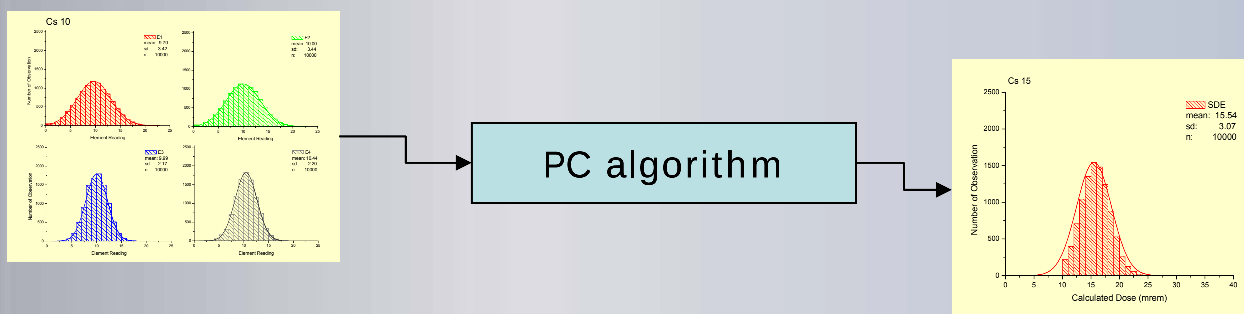
$$u_c^2(y) = \sum_{i=1}^{N}\left(\frac{\partial f}{\partial x_i}\right)^2 u^2(x_i) + 2\sum_{i=1}^{N-1}\sum_{j=i+1}^{N}\frac{\partial f}{\partial x_i}\frac{\partial f}{\partial x_j}u(x_i, x_j). \quad \text{(A-3)}$$

Pros – Real-time calculation. Results generated for all calculated doses.

Cons – Difficult with complicated algorithms, complications with co-variance term. What about decision points?

Total Propagated Uncertainty
What about the algorithm?

Option 2 - Generate sample of results and process through algorithm and analyze resultant distribution (GUM Supplement 1)



Pros – Result is as good as the input. Can be generated for any algorithm.

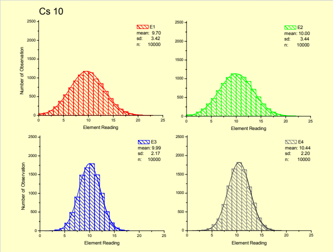
Cons – Must be run for each dose and field condition.

Process Overview

Identify sources
of uncertainty in element readings

Calculate
total combined uncertainty
in element readings

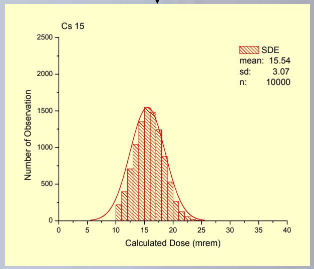
Generate file of
input responses (GUM Suppl. 1)



Process
using PC algorithm

PC algorithm

Analyze output
distribution of doses



GUM Supplement 1

Appendix C

Generate "random" samples from assumed distribution using Box-Muller Gaussian pseudo-random number generator.

1. *Generate rectangular random variates between 0 and 1. Excel function* RAND()
2. *Calculate two "draws", Z_1 and Z_2, from Gaussian variable with mean=0, sd = 1*
3. *Apply to assumed distribution by:*
   $X = \mu + \sigma \cdot Z$

Input parameters	
None	
Output parameters	
Z_1, Z_2	Two draws from a Gaussian variable with zero expectation and unit standard deviation
Computation	
	a) Generate rectangular random variates V_1 and V_2 between zero and one b) Form $Z_1 = \sqrt{-2 \log V_1} \cos 2\pi V_2$ and $Z_2 = \sqrt{-2 \log V_1} \sin 2\pi V_2$ c) Take Z_1 and Z_2 as *two* standard Gaussian variates

Table C.3 — The Box-Muller Gaussian pseudo-random number generator.

Generated Input Distributions

Simple example – Panasonic 802

100 mrem ^{137}Cs

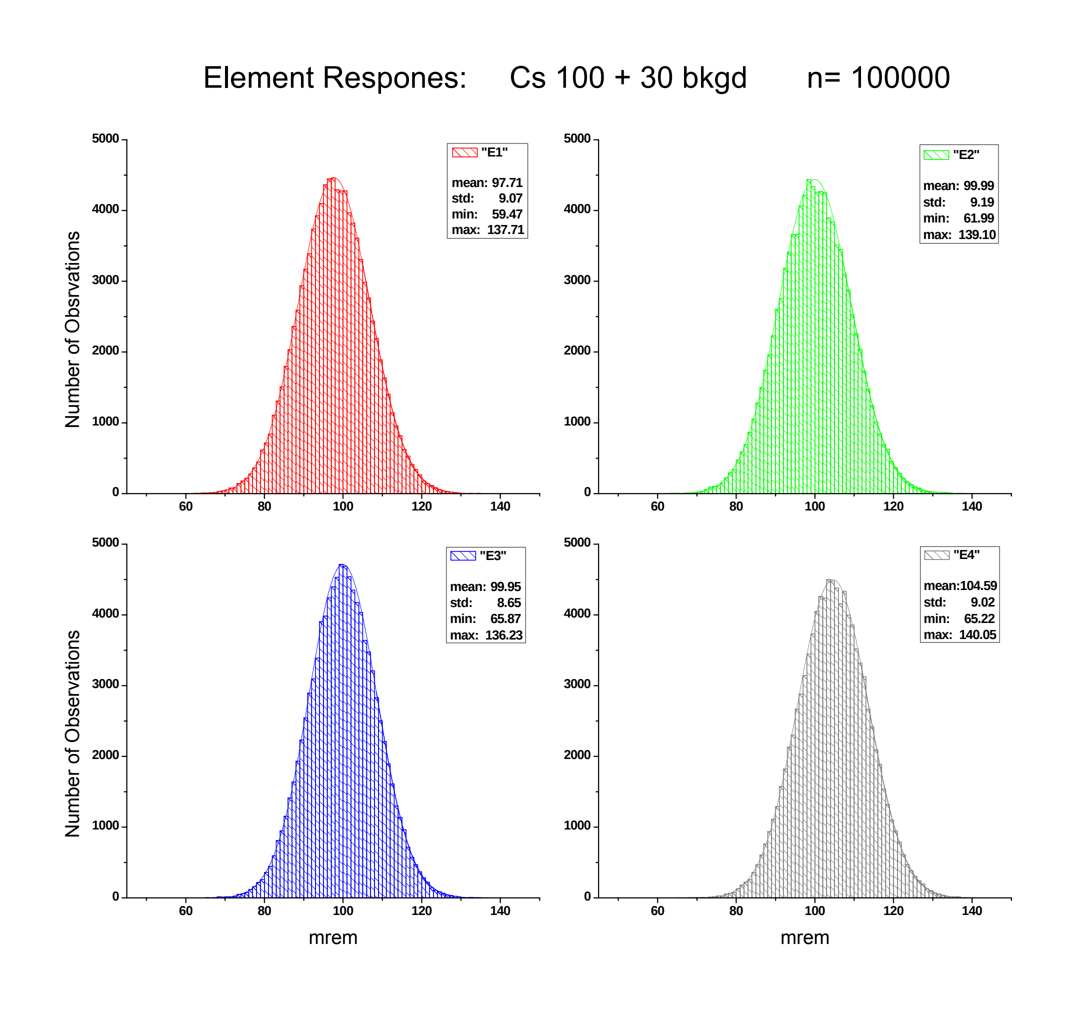
+ 30 mrem bkgd (Cs)

Calculated statistics for *net responses*:

	E1	E2	E3	E4
μ	97	100	100	100
σ	8.99	9.22	8.67	8.67

Generated Input Distributions

Simple example - continued

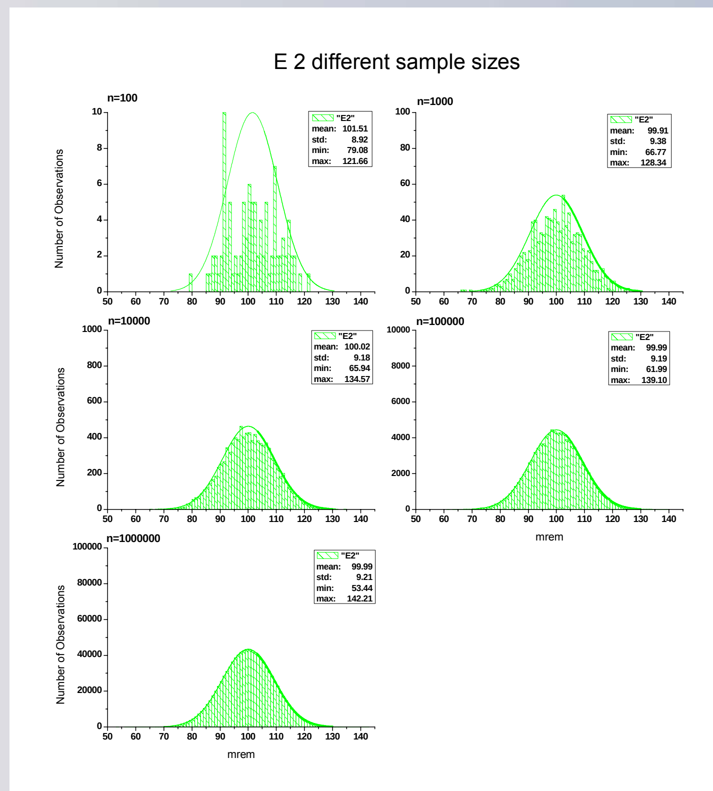


Generated Input Distributions

Simple example - continued Effect of sample size

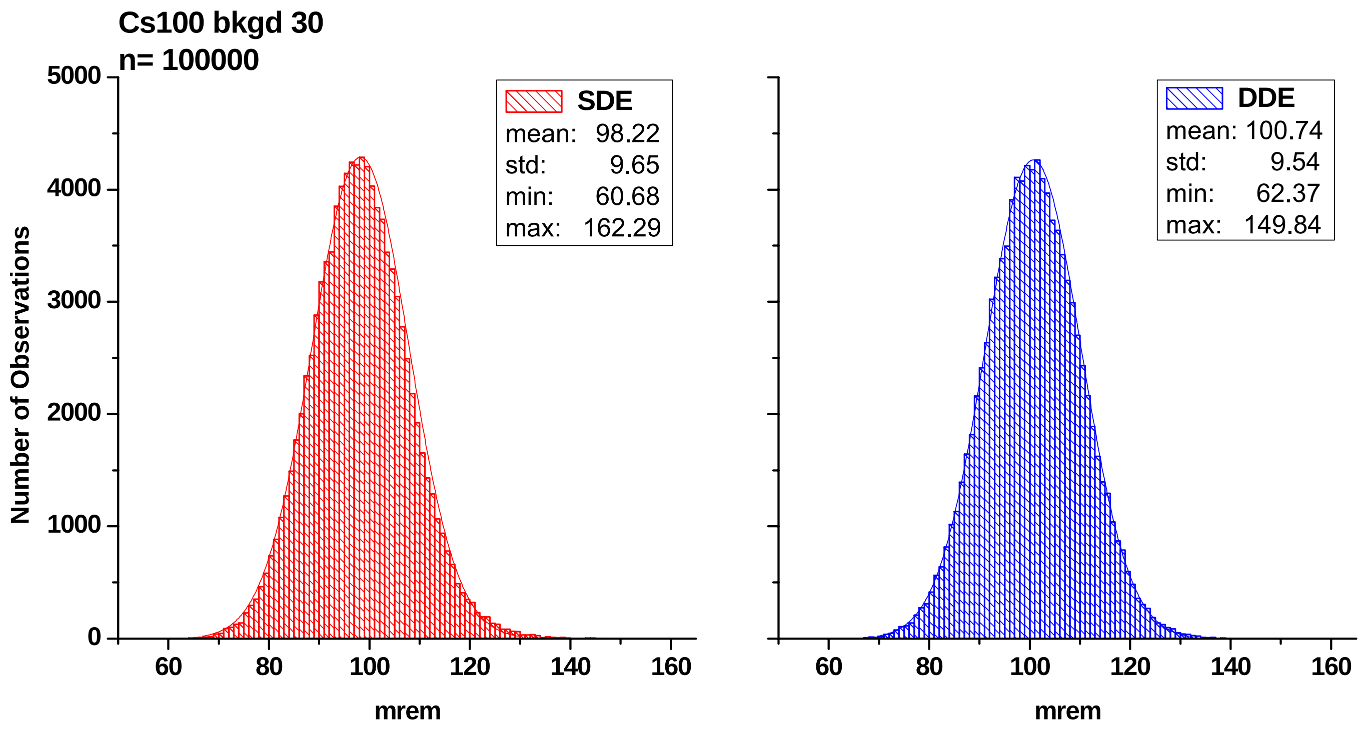
E2 response for 100 mrem ^{137}Cs with 30 mrem background

Calculated	**9.22**
N=100	8.92
N=1000	9.38
N=10000	9.18
N=100000	9.20
N=1000000	9.22



Time to generate 100k sample: < 10 sec

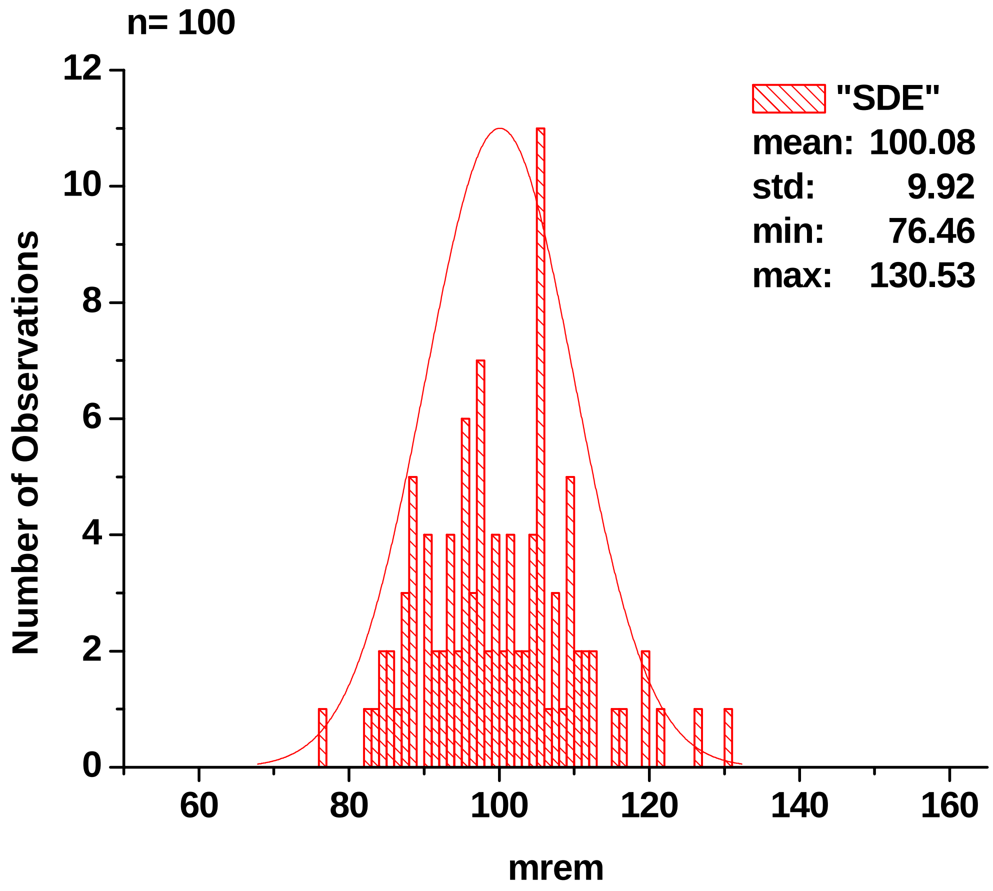
Output file



Time to process 100k output: < 20 min; 1M < 2 hrs

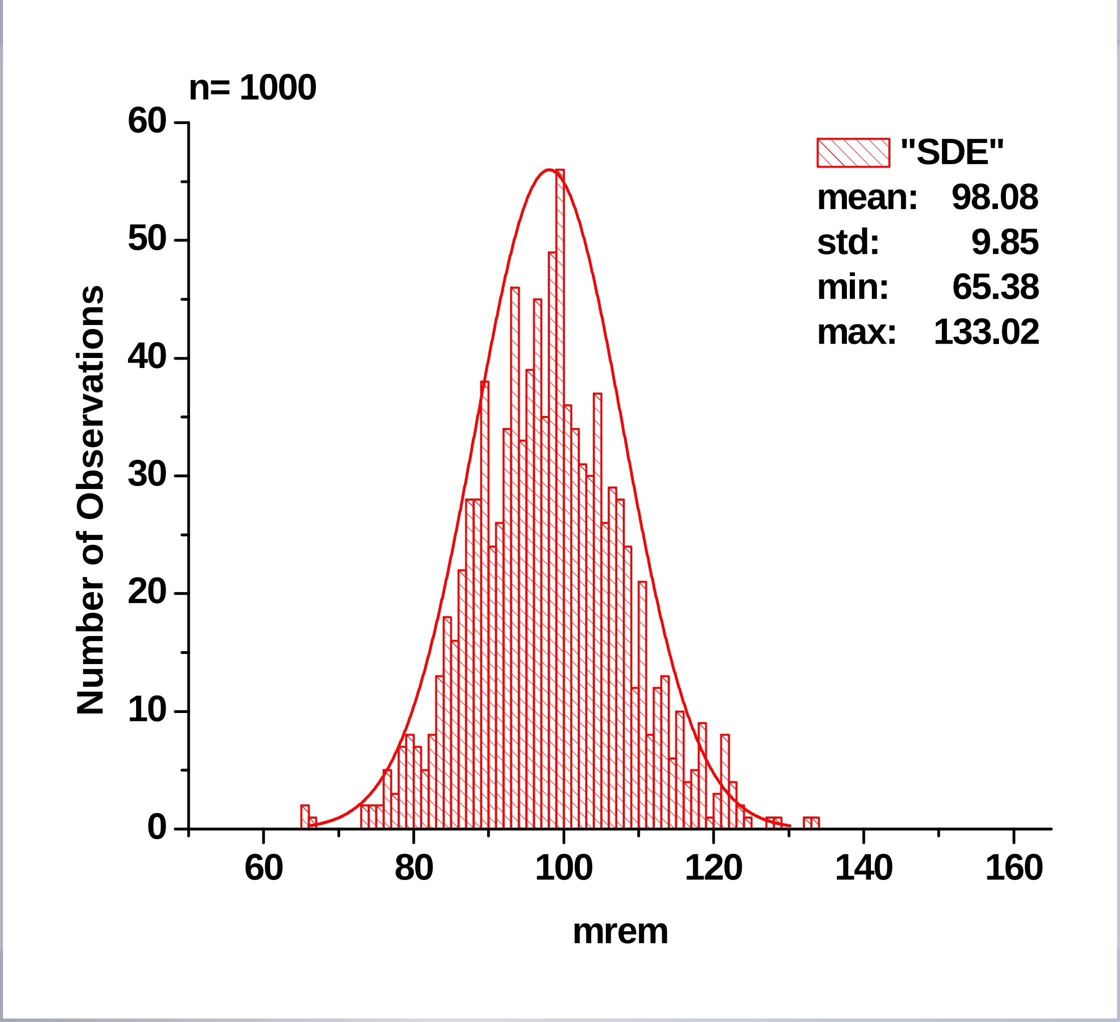
Effect of n on output distribution

100 mrem ^{137}Cs, 30 mrem bkgd



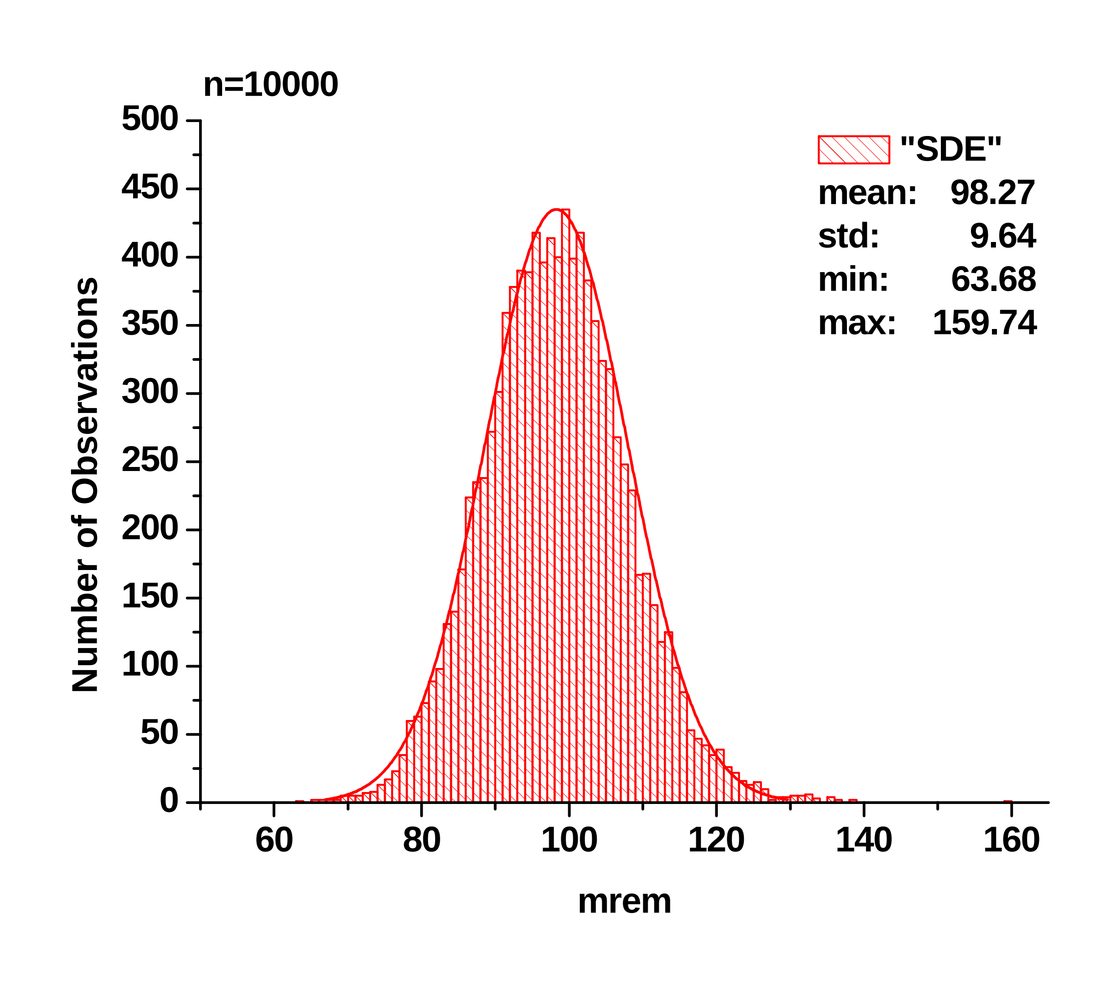
Effect of n on output distribution

100 mrem ^{137}Cs, 30 mrem bkgd



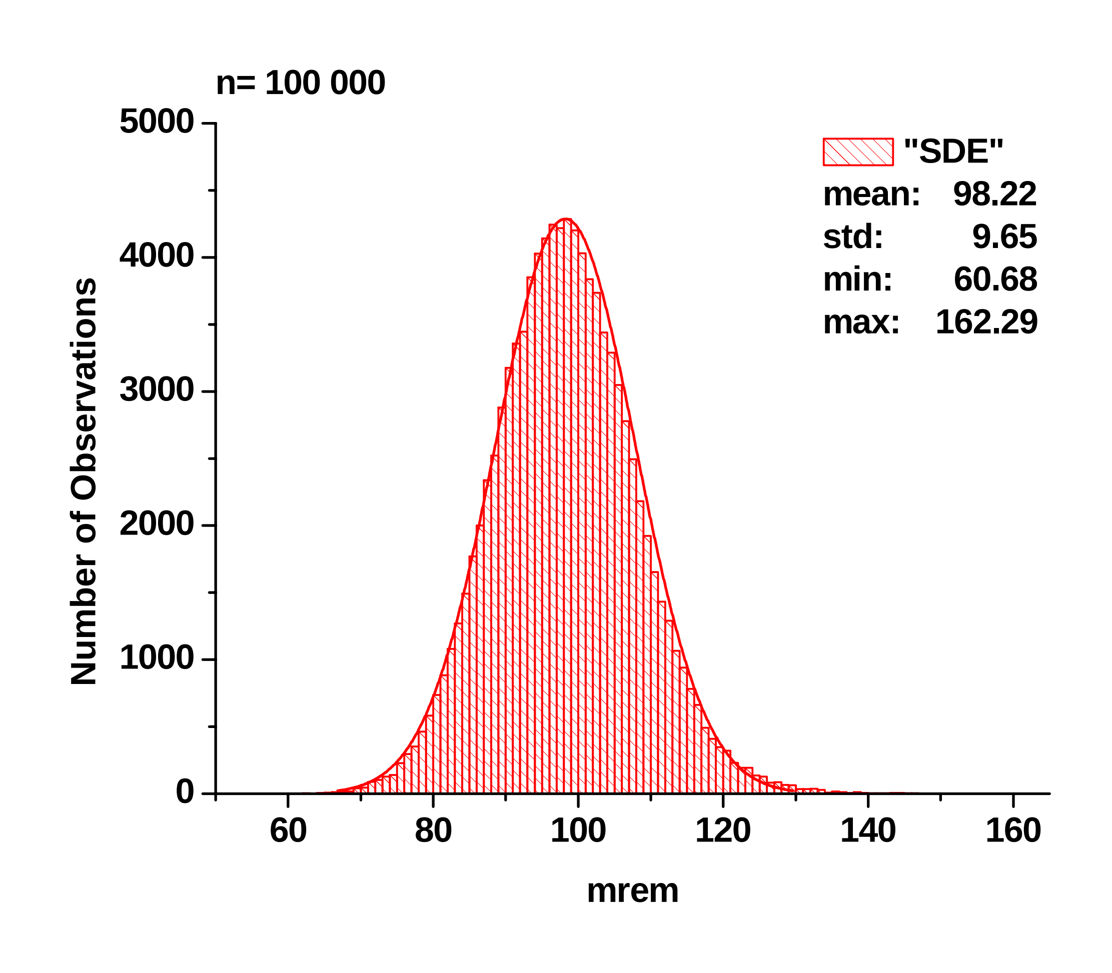
Effect of n on output distribution

100 mrem ^{137}Cs, 30 mrem bkgd



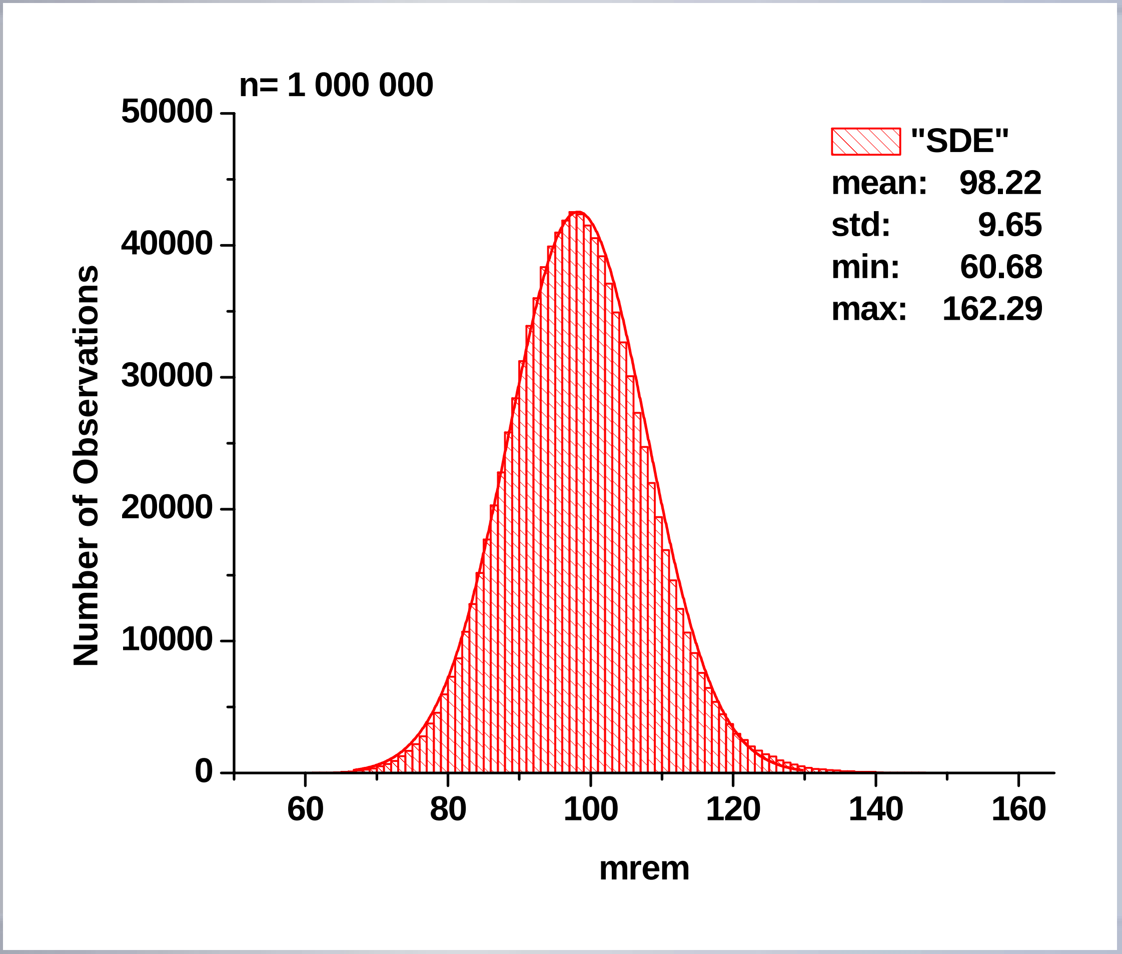
Effect of n on output distribution

100 mrem ^{137}Cs, 30 mrem bkgd



Effect of n on output distribution

100 mrem ^{137}Cs, 30 mrem bkgd



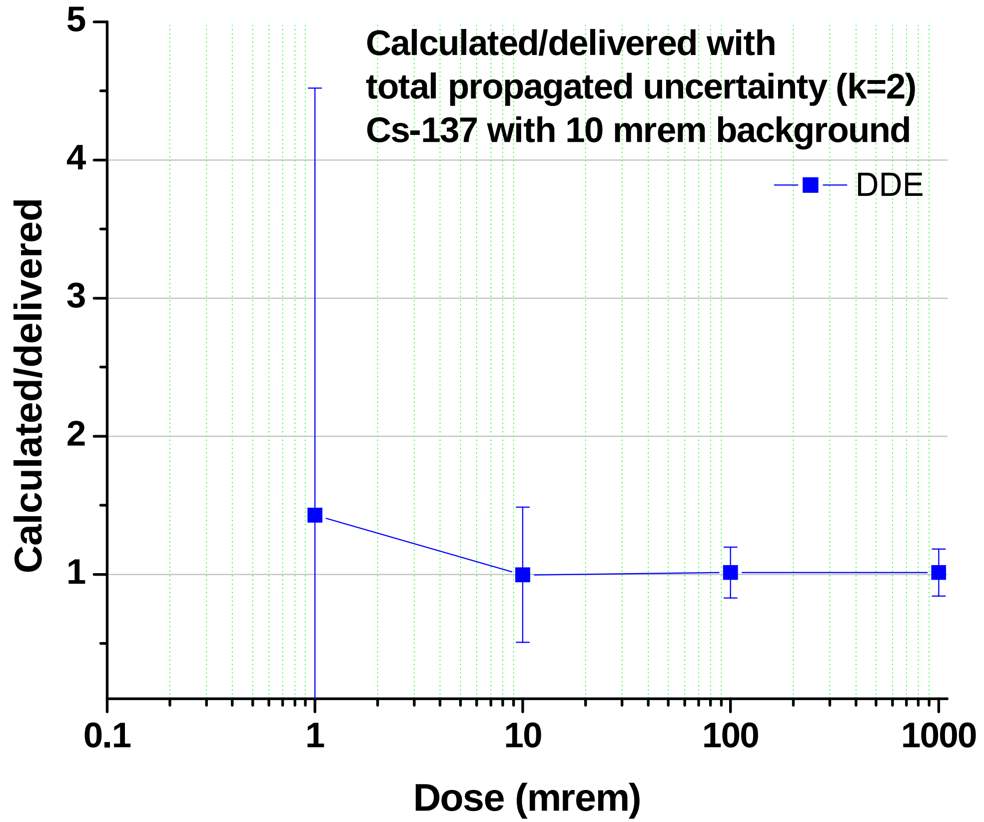
Application

- Uncertainty vs dose
- Algorithm test points
- Mixture performance

Uncertainty vs. dose
^{137}Cs with 10 mrem bkgd

Results of four different runs with doses of 1, 10, 100, and 1000 mrem ^{137}Cs.

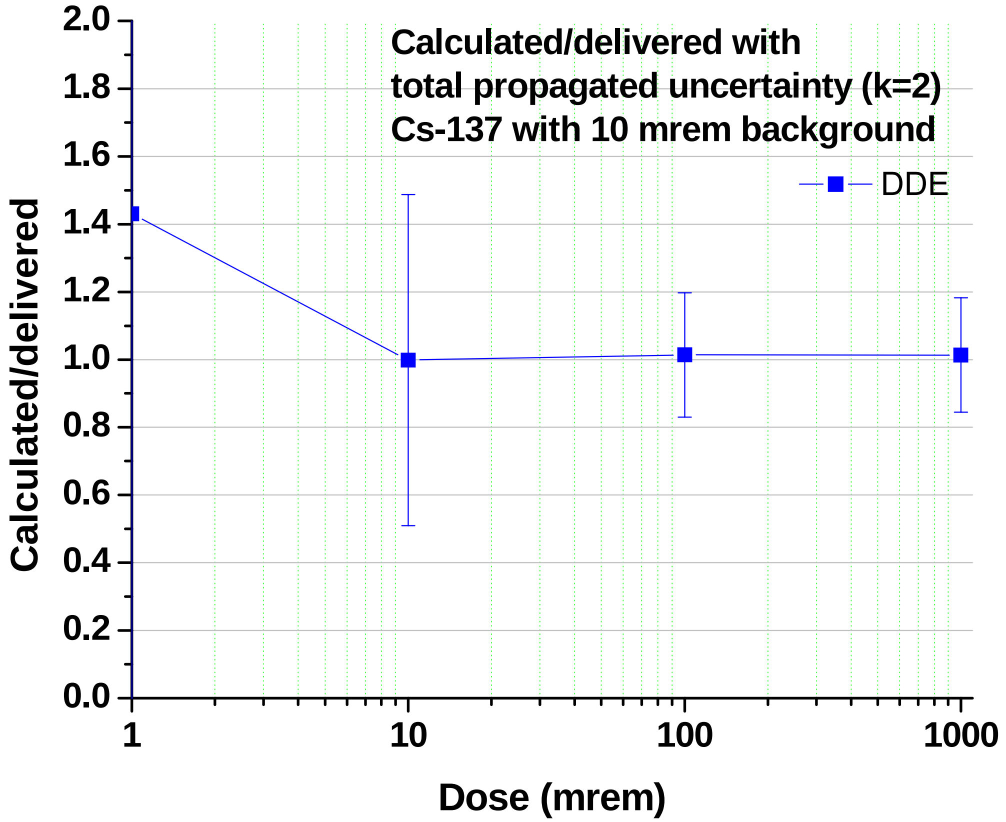
Two standard deviation error bars (coverage factor = 2)



Uncertainty vs. dose
^{137}Cs with 10 mrem bkgd - detail

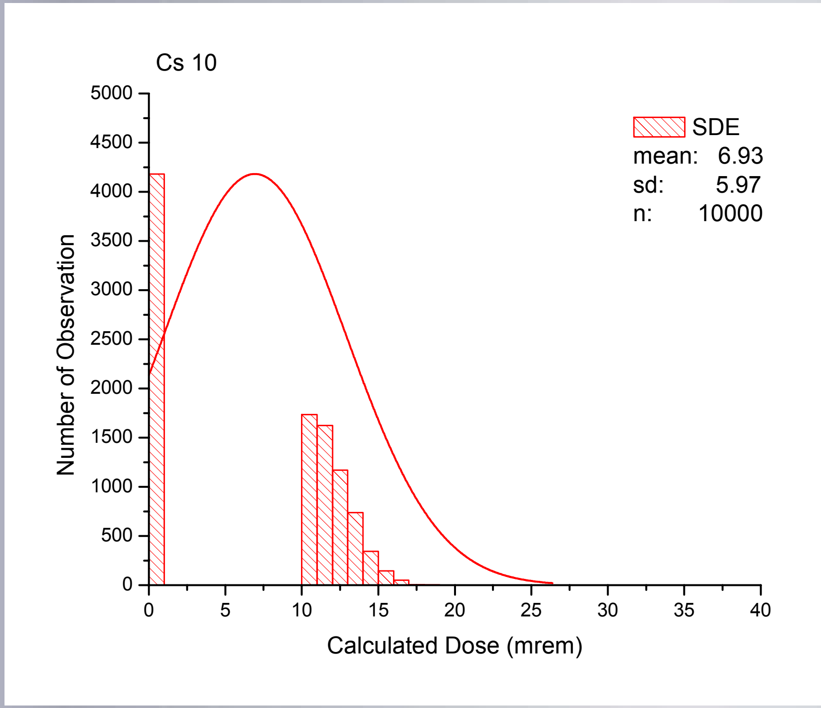
Results of four different runs with doses of 1, 10, 100, and 1000 mrem ^{137}Cs.

Two standard deviation error bars (coverage factor = 2)

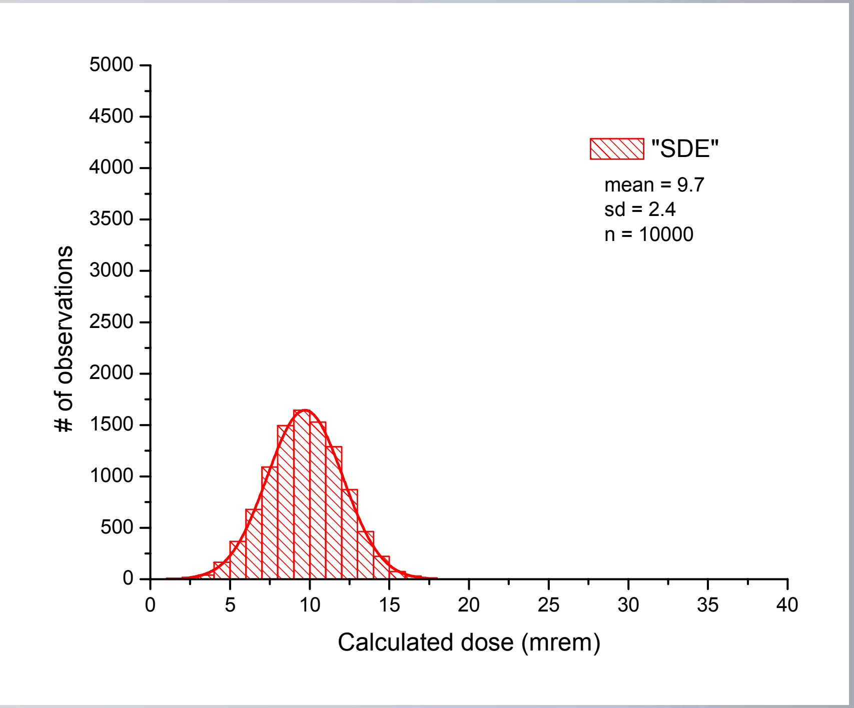


Algorithm Decision Points

Minimum reportable = 10



Minimum reportable = 0



Mixture Performance

30 mrem ^{137}Cs + 100 mrem ^{204}Tl (10 bkgd)

Expected responses:

Field	SDE	DDE	E1	E2	E3	E4
^{204}Tl	100	0	70	0	0	0
^{137}Cs	30	30	29	30	30	30
Total	**130**	**30**	**99**	**30**	**30**	**30**
s.d. (est)			**8.6**	**3.6**	**2.6**	**2.6**

Enter Parameters

File Help

Stanford Dosimetry Uncertainty Propagation Tool 12/10/05

Description: Cs 30 Tl 100 bkgd 10

Dose: 100

E1	E2	E3	E4
.970	1	1	1

	Element 1	Element 2	Element 3	Element4
Background	10	10	10	10
Net reading	98.7	30	30	30
SigmaNet	5.08	2.91	1.58	1.58

Sources of uncertainty

Parameter file: ...\parameters.txt

Load parameters

Save parameters

		Contribution	%	Contribution	%	Contribution	%	Contribution	%
RCF sd	EDIT	0.	0.00%	0.	0.00%	0.	0.00%	0.	0.00%
Linearity	EDIT	5.7	5.77%	1.7	5.77%	1.7	5.77%	1.7	5.77%
Fade	EDIT	2.6	2.60%	0.8	2.60%	0.8	2.60%	0.8	2.60%
Calibration bias	EDIT	3.	3.00%	0.9	3.00%	0.9	3.00%	0.9	3.00%

Update

Make File

Net Response	98.7	30.	30.	30.
Combined Uncertainty	8.58	3.59	2.63	2.63
As a percentage	8.69%	11.97%	8.76%	8.76%

Quit

Mixture Performance

30 mrem ^{137}Cs + 100 mrem ^{204}Tl (10 bkgd)

	Expected	Alg. results	|----------------Observed	---------------|		
			mean	sd	min	max
E1	98.7		98.6	8.6	60.6	140.0
E2	30		30.0	3.6	14.8	45.1
E3	30		30.0	2.6	18.9	41.5
E4	30		30.2	2.6	18.1	41.5
SDE	130	**129.3**	**111.3**	39.4	14.8	185.1
DDE	30	**29.9**	**29.8**	3.1	15.3	41.9

Mixture Performance

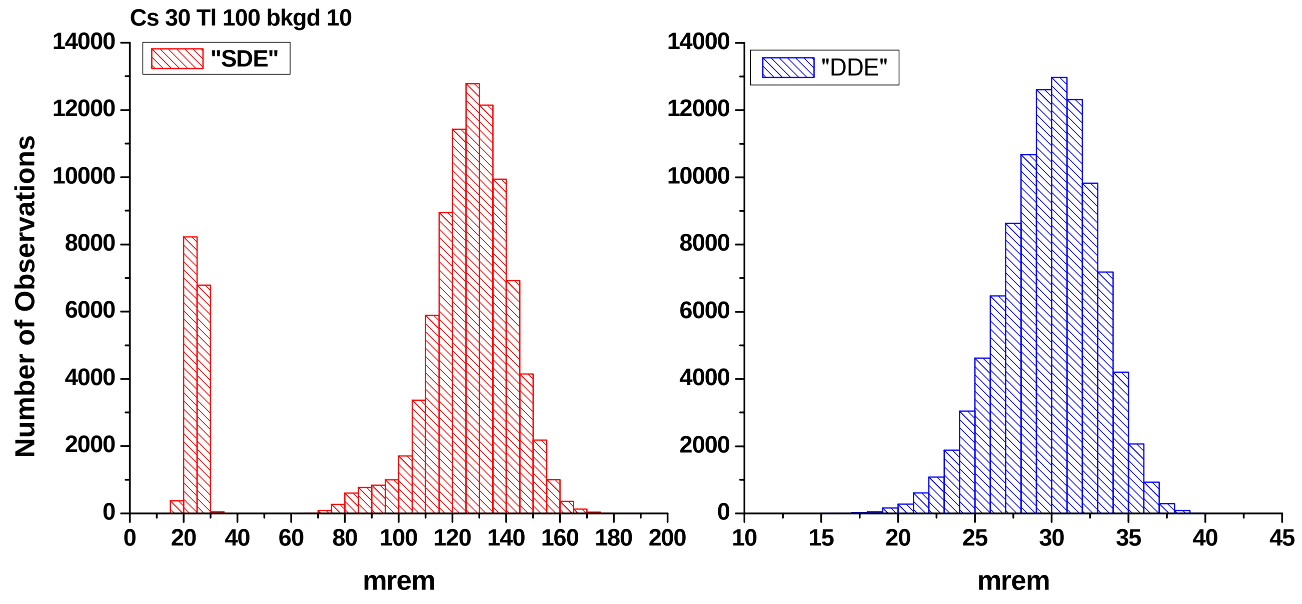
30 mrem ^{137}Cs + 100 mrem ^{204}Tl (10 bkgd)

Bimodal (at least) distribution for SDE

17% missed the beta dose

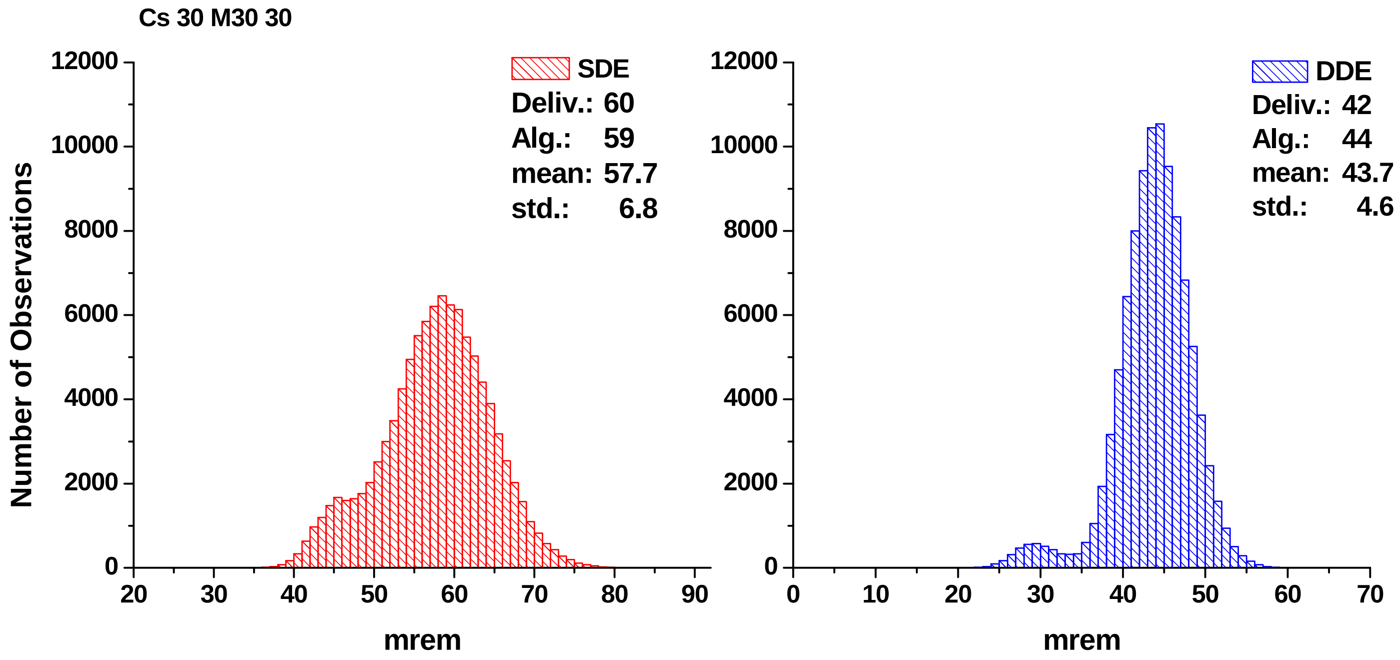
80% of the doses are ± 20% of the "true" dose

For DDE 95% are between 23-36



Mixture Performance

30 mrem ^{137}Cs + 30 mrem M30 (10 bkgd)



Mixture Performance

30 mrem M30 + 150mrem ^{204}Tl (30 bkgd)

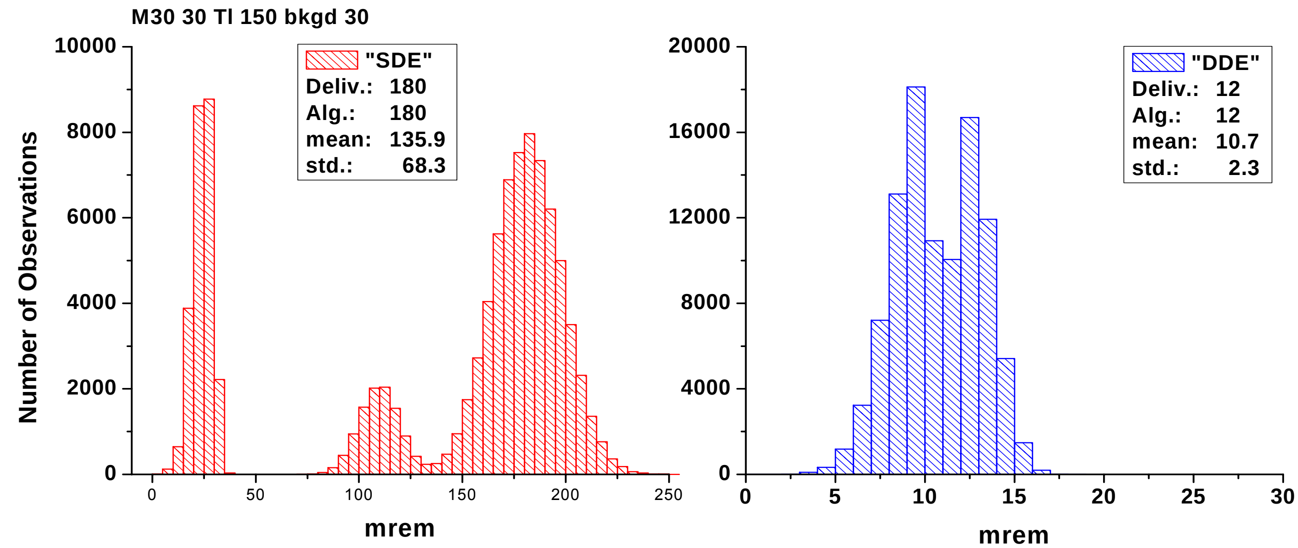
Multi-modal distributions

25% missed the beta dose

10% saw the beta as high energy

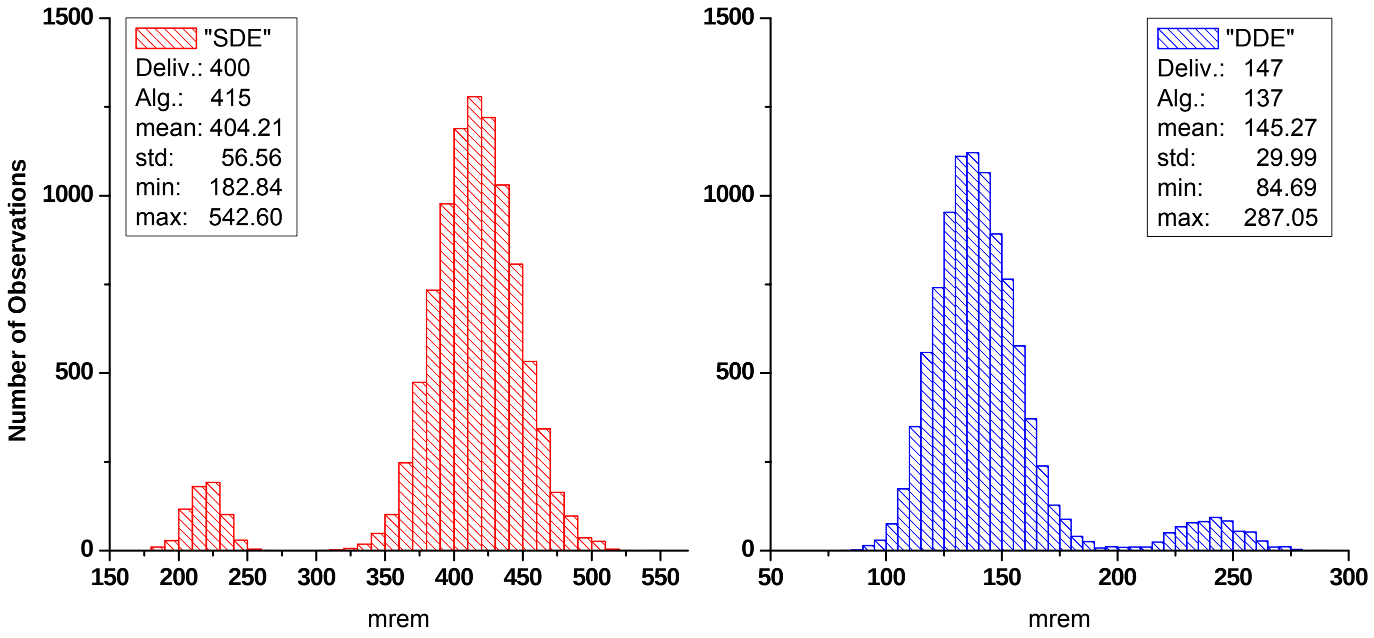
65% of the SDE doses are ± 20% of the “true” dose

75% of DDE ± 20% of true dose



Mixture Performance

50mrem S60+ 100mrem ^{137}Cs + 250mrem ^{90}Sr/^{90}Y (10mrem bkgd)



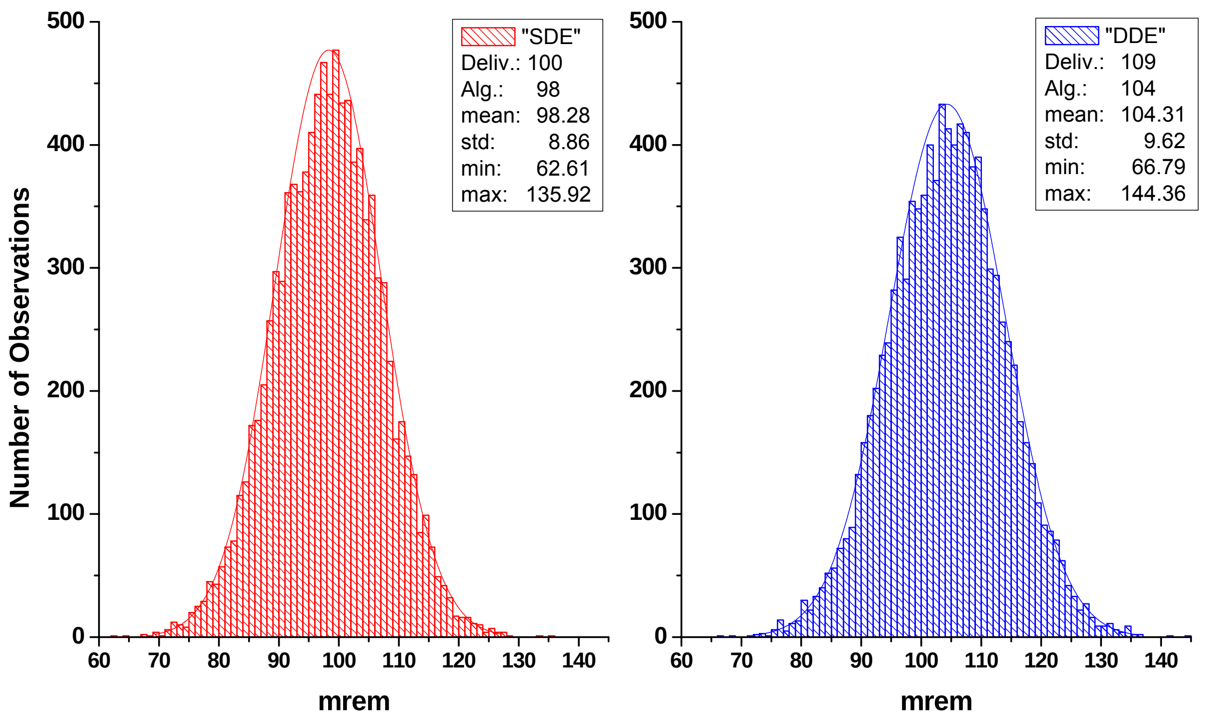
Mixture Performance
100 mrem M150 60 deg horiz. (10 bkgd)

Expected results for
M150 at 60 degrees horizontal

- E1: 80.1
- E2: 80.9
- E3: 638
- E4: 281
- SDE = 100
- DDE = 109

Mixture Performance

100 mrem M150 60 deg horiz. (10 bkgd)



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